

Model for the Interaction of Thermal Radiation with a Monatomic Plasma

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A kinetic model for a weakly-ionized plasma interacting with radiation is proposed based on the energy-momentum method, in which both energy and momentum of the system are conserved. The radiation-initiated ionization is obtained in a non-inertial coordinate system, and the proposed model satisfies the H-theorem.

1. Introduction

The advent of laser technology and possibility of laser-driven thermonuclear fusion¹ has generated interest in the problem of a plasma interacting with thermal radiation. The laser radiation plays quite a varied role, and radiative forces have been known to contribute to phenomena such as plasma acceleration², self-focusing of laser radiation³, magnetic field generation⁴, and parametric instabilities⁵.

Recently, kinetic models for the interaction of thermal radiation with a rarefied diatomic gas have been proposed by Phillips and Arpaci⁶ and Saraf^{7–9}. Though the coupling of radiation field with an ionized gas has been widely investigated from the continuum viewpoint¹⁰, there have recently been two attempts by Anderson, Baum and Krook¹¹, and by Phillips and Arpaci¹² to study this coupling at the level of kinetic theory¹³. The energy method^{6,7} used by Phillips and Arpaci¹² conserves the total energy of isolated system of plasma and radiation but completely ignores the contributions of the particle and radiative momentums. However, the exchange of momentum between the radiation field and plasma is important in laser-induced fusion wherein intense optical radiation is absorbed by the plasma. In addition, Phillips and Arpaci¹² have neglected the effect of macroscopic motion of plasma on radiation so that the radiative transfer equation applies only to an inertial coordinate system. The Boltzmann equations for material particles are conveniently written in a non-inertial frame and moreover, in such a frame the absorption coefficient of radiation is usually independent of the direction of

propagation of photons. Hence, Saraf^{7,8} used radiative transport equations¹³ in a non-inertial system to formulate models for the interaction of radiation with a diatomic gas.

In this paper, we use the energy-momentum method⁸ to model a weakly ionized plasma interacting with radiation. The radiative transfer equation in non-inertial frame is employed for obtaining model equations for photon-initiated ionization.

2. Formulation

The analysis in this study is based on all the assumptions listed in § 2 of Phillips and Arpaci¹², with the exception of (V). We consider a weakly-ionized three-component plasma comprised of electrons, ions and atoms which are assumed to be monatomic particles. A non-equilibrium degree of ionization is defined as

$$\Phi = (n_i n_e / n^2)^{1/2}, \quad (1)$$

where n_i , n_e and n denote, respectively, the number density of ions, electrons and heavy particles (atoms plus ions). In equilibrium, $n_i = n_e = n_{e0}$ and the equilibrium degree of ionization $\Phi_0 = n_{e0}/n$ in the special case of weak (Φ , $\Phi_0 \ll 1$) one-fold ionization can be shown¹² to be

$$\Phi_0 \cong \left(\frac{T}{\Theta_e} \right)^{5/4} \left(\frac{p_0}{p} \right)^{1/2} \left(\frac{Z_e^i}{Z_e^a} \right)^{1/2} \exp \{ -\Theta_1/2 T \}, \quad (2)$$

where p is the pressure, T is the temperature, Θ_e is electronic temperature, p_0 is the standard atmospheric pressure, $\Theta_1 (= \varepsilon_1/k)$ is the characteristic temperature for ionization, k is Boltzmann constant, ε_1 is the ionization potential, and Z_e^i and Z_e^a are respectively, the electronic partition functions for the ion and atom.

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Assuming ideal gas law $p = n(1 + \Phi_0)kT$, we have

$$\left(\frac{\partial \Phi_0}{\partial T}\right)_n = \frac{1}{2} \left(\frac{3}{2} + \frac{\Theta_I}{T}\right) \frac{\Phi_0}{T}. \quad (3)$$

In terms of internal energy $u_0 = \frac{3}{2}(1 + \Phi_0)R_a T + \Phi_0 R_a \Theta_I$, the specific heat at constant volume is

$$C_v = \left(\frac{\partial u_0}{\partial T}\right)_n = \frac{3}{2}(1 + \Phi_0)R_a + \frac{1}{2} \left(\frac{3}{2} + \frac{\Theta_I}{T}\right)^2 \Phi_0 R_a, \quad (4)$$

where R_a is gas constant for the atom.

Denoting by $f^r(\mathbf{x}, \mathbf{V}, t)$ the velocity distribution functions, where \mathbf{V} is particle velocity, the mean velocity $\langle \mathbf{V} \rangle_r$ and temperature T_r of the r -species are defined as

$$n_r \langle \mathbf{V} \rangle_r = \int n_r f^r d\mathbf{V} d\Omega, \quad (5)$$

and

$$3k n_r T_r = \int m_r \mathbf{U}_r^2 n_r f^r d\mathbf{V} d\Omega \equiv m_r n_r \langle \mathbf{U}_r^2 \rangle_r, \quad (6)$$

where

$$\mathbf{U}_r = \mathbf{V} - \langle \mathbf{V} \rangle_r, \quad (7)$$

is the random thermal velocity of the r -species, and m_r is mass of an r -particle.

The mean mass, or drift velocity of the plasma is defined as

$$\bar{\mathbf{V}} = \sum_r m_r n_r \langle \mathbf{V} \rangle_r / \sum_r m_r n_r. \quad (8)$$

The thermal velocity is defined as

$$\mathbf{U} = \mathbf{V} - \bar{\mathbf{V}}. \quad (9)$$

The model kinetic equations for plasma-radiation interaction proposed by Phillips and Arpaci¹² are

$$\begin{aligned} \frac{\partial}{\partial t} (n_r f^r) + \mathbf{V} \cdot \frac{\partial}{\partial \mathbf{x}} (n_r f^r) + \frac{\mathbf{F}^r}{m_r} \cdot \frac{\partial}{\partial \mathbf{V}} (n_r f^r) \\ = \sum_s \left[\frac{\delta}{\delta t} (n_r f^r) \right]_s + \left[\frac{\delta}{\delta t} (n_r f^r) \right]_M^I + \left[\frac{\delta}{\delta t} (n_r f^r) \right]_P^I, \end{aligned} \quad (10)$$

where \mathbf{F}^r is applied force, and the indices r, s apply to electrons, ions and atoms.

A) Elastic Collisions

The summation term on rhs of (10) represents the elastic self- and cross-collisions between the electrons, ions, and atoms, and could be written as

$$\sum_s \left[\frac{\delta}{\delta t} (n_r f^r) \right]_s = \sum_s n_r n_s \sigma_{rs} \langle \mathbf{U} \rangle_{rs} (f_{0s}^r - f^r), \quad (11)$$

where σ_{rs} is the collision cross-section, and

$$f_{0s}^r = \left(\frac{m_r}{2\pi k T_{rs}} \right)^{3/2} \exp \left[-\frac{m_r}{2k T_{rs}} (\mathbf{V} - \langle \mathbf{V} \rangle_{rs})^2 \right], \quad (12)$$

$$\langle \mathbf{U} \rangle_{rs} = [(\langle \mathbf{U} \rangle_r)^2 + (\langle \mathbf{U} \rangle_s)^2]^{1/2}. \quad (13)$$

The quantities $\langle \mathbf{V} \rangle_{rs}$ and T_{rs} are well-known to be

$$\langle \mathbf{V} \rangle_{rs} = \frac{m_r \langle \mathbf{V} \rangle_r + m_s \langle \mathbf{V} \rangle_s}{m_r + m_s}, \quad (14)$$

and

$$T_{rs} = \frac{1}{6k} \left[m_r \langle \mathbf{U}_r^2 \rangle_r + m_s \langle \mathbf{U}_s^2 \rangle_s + \frac{m_r m_s}{m_r + m_s} (\langle \mathbf{V} \rangle_r - \langle \mathbf{V} \rangle_s)^2 \right]. \quad (15)$$

We note that for $r=s$, $\langle \mathbf{V} \rangle_{rr} = \langle \mathbf{V} \rangle_r$, $T_{rr} = T_r$ so that f_{0r}^r is the Maxwellian distribution for r -species at temperature T_r and mean velocity $\langle \mathbf{V} \rangle_r$, given by

$$f_{0r}^r = \left(\frac{m_r}{2\pi k T_r} \right)^{3/2} \exp \{ -m_r \mathbf{U}_r^2 / 2k T_r \} \quad (16)$$

where \mathbf{U}_r is defined by (7).

B) Particle Ionization

Phillips and Arpaci¹² noted that the depleted ions and electrons are associated with the replenished atoms, and vice versa. The particle-particle ionization represented by second term on rhs of (10) can be modelled as¹²

$$\left[\frac{\delta}{\delta t} (n_a f^a) \right]_M^I = n^2 \sigma_M^I \langle \mathbf{U} \rangle_{ei} (\Phi_0^2 f_0^a - \Phi_0^2 f^a), \quad (17)$$

$$\left[\frac{\delta}{\delta t} (n_i f^i) \right]_M^I = n^2 \sigma_M^I \langle \mathbf{U} \rangle_{ei} (\Phi_0^2 f_0^i - \Phi_0^2 f^i), \quad (18)$$

and

$$\left[\frac{\delta}{\delta t} (n_e f^e) \right]_M^I = n^2 \sigma_M^I \langle \mathbf{U} \rangle_{ei} (\Phi_0^2 f_0^e - \Phi_0^2 f^e), \quad (19)$$

where σ_M^I denotes the collision cross-section for the particle-initiated ionization, and

$$f_0^r = \left(\frac{m_r}{2\pi k T_M^I} \right)^{3/2} \exp [-m_r (\mathbf{V} - \langle \mathbf{V} \rangle_M^I)^2 / (2k T_M^I)], \quad (20)$$

$$\begin{aligned} \left[1 + \left(\frac{\Phi}{\Phi_0} \right)^2 \right] \langle \mathbf{V} \rangle_M^I = \langle \mathbf{V} \rangle_a \\ + \left(\frac{\Phi}{\Phi_0} \right)^2 \left[\frac{m_e}{m_a} \langle \mathbf{V} \rangle_e + \frac{m_i}{m_a} \langle \mathbf{V} \rangle_i \right], \end{aligned} \quad (21)$$

and

$$\begin{aligned} \left[1 + \left(\frac{\Phi}{\Phi_0} \right)^2 \right] T_M^I \cong T_a + \left(\frac{\Phi}{\Phi_0} \right)^2 (T_e + T_i) \\ + \frac{2}{3} \Theta_I \left[\left(\frac{\Phi}{\Phi_0} \right)^2 - 1 \right]. \end{aligned} \quad (22)$$

C) Radiative Transfer

The radiation is taken to have a continuous spectrum in visible or ultra-violet region, resulting from bound-free transitions. Since heavy particles are assumed to remain in their electronic ground state, the line radiation from bound-bound transitions is absent. Also, we neglect the photon-photon scattering as well as the continuous radiation related to electron-photon interaction. The evolution of specific intensity I_ν of radiation is given by the radiative transfer equation

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \mathbf{l} \cdot \frac{\partial I_\nu}{\partial \mathbf{x}} = K_\nu (I_{\nu 0} - I_\nu), \quad (23)$$

where c is speed of light, ν is spectral frequency, \mathbf{l} is the propagation direction, K_ν is the coefficient of monochromatic absorption, and $I_{\nu 0}$ is the equilibrium Planck distribution at "local" temperature T_e for electron-species:

$$I_{\nu 0} = \frac{2 h \nu^3}{c^2} [\exp \{h \nu / k T_e\} - 1]^{-1}, \quad (24)$$

where h is Planck's constant.

Finally, we note that the last term on rhs of (10) represents radiation-initiated ionization and we now turn to model this term.

3. Photon Ionization by Energy-Momentum Method⁸

As an electron is very much lighter than an ion or an atom, the photons mainly collide with electrons in the plasma, and thus the radiative interactions do not appreciably change the momentum and kinetic energy of the heavy particles. Sampson¹³ has obtained transport equations for radiative momentum and energy in non-inertial frames of reference. Now, according to the energy-momentum

principle⁸, the radiative momentum is equated to the rate of change in momentum of electrons, while the radiative energy is balanced by the rate of change in the energy of electrons and with the ionization of atoms. Thus, we have

$$\left[\frac{\partial}{\partial t} (m_e n_e \langle \mathbf{V} \rangle_e) \right]_P^I = \frac{1}{c} \int K_\nu \mathbf{l} (I_\nu - I_{\nu 0}) d\nu d\Omega, \quad (25)$$

and

$$\left[\frac{\partial}{\partial t} \left(\frac{1}{2} m_e n_e \langle \mathbf{U}_e^2 \rangle_e \right) \right]_P^I + \epsilon_I \left[\frac{\partial n_i}{\partial t} \right]_P^I = \int K_\nu \left(I_\nu - I_{\nu 0} - \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega. \quad (26)$$

We can rearrange the last term in (10) as

$$\left[\frac{\partial}{\partial t} (n_e f_e^e) \right]_P^I = \frac{\partial (n_e f_e^e)}{\partial (m_e n_e \langle \mathbf{V} \rangle_e)} \left[\frac{\partial}{\partial t} (m_e n_e \langle \mathbf{V} \rangle_e) \right]_P^I + \frac{\partial (n_e f_e^e)}{\partial (\frac{1}{2} m_e n_e \langle \mathbf{U}_e^2 \rangle_e)} \left[\frac{\partial}{\partial t} \left(\frac{1}{2} m_e n_e \langle \mathbf{U}_e^2 \rangle_e \right) \right]_P^I. \quad (27)$$

As the mean free path of a photon is much larger than that of an electron, we can take electrons to be distributed according to Maxwellian equilibrium f_{0e}^e [given by (16)] during photon relaxation, and write

$$\frac{\partial (n_e f_e^e)}{\partial (m_e n_e \langle \mathbf{V} \rangle_e)} = \frac{\partial (n_{e0} f_{0e}^e)}{\partial (m_e n_{e0} \langle \mathbf{V} \rangle_e)}, \quad (28)$$

and

$$\frac{\partial (n_e f_e^e)}{\partial (\frac{1}{2} m_e n_e \langle \mathbf{U}_e^2 \rangle_e)} = \frac{\partial (n_{e0} f_{0e}^e) / \partial T_e}{\partial (\frac{1}{2} m_e n_{e0} \langle \mathbf{U}_e^2 \rangle_e) / \partial T_e}. \quad (29)$$

But, from (6), (3) and (16), we have

$$m_e \langle \mathbf{U}_e^2 \rangle_e = 3 k T_e, \quad (30)$$

$$\frac{\partial n_{e0}}{\partial T_e} = \frac{1}{2} \left(\frac{3}{2} + \frac{\Theta_I}{T_e} \right) \frac{n_{e0}}{T_e}, \quad (31)$$

and

$$\frac{\partial f_{0e}^e}{\partial T_e} = \left(\frac{m_e \mathbf{U}_e^2}{2 k T_e} - \frac{3}{2} \right) \frac{f_{0e}^e}{T_e}. \quad (32)$$

Thus, we can rewrite (27) as

$$\left[\frac{\partial}{\partial t} (n_e f_e^e) \right]_P^I = \frac{\mathbf{U}_e f_{0e}^e}{c k T_e} \cdot \int K_\nu \mathbf{l} (I_\nu - I_{\nu 0}) d\nu d\Omega + \frac{(3 k T_e - 2 \epsilon_I - 2 m_e \mathbf{U}_e^2) f_{0e}^e}{3 k T_e (\epsilon_I + 7 k T_e / 2)} \left\{ \epsilon_I \left[\frac{\partial n_i}{\partial t} \right]_P^I + \int K_\nu \left(I_{\nu 0} - I_\nu - \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega \right\}. \quad (33)$$

Since charge is conserved, we have

$$\left[\frac{\partial n_i}{\partial t} \right]_P^I = \left[\frac{\partial n_e}{\partial t} \right]_P^I = \int \left[\frac{\partial}{\partial t} (n_e f_e^e) \right]_P^I d\Omega d\mathbf{V} = \frac{(3 k T_e + 2 \epsilon_I) R_a \Phi_0}{4 (k T_e)^2 C_V^I} \int K_\nu \left(I_\nu - I_{\nu 0} + \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega, \quad (34)$$

where

$$C_V^I = C_V - 3 R_a / 2.$$

And, as the ionization does not change the number density of heavy particles, we have

$$\left[\frac{\delta n_a}{\delta t} \right]_P^I + \left[\frac{\delta n_i}{\delta t} \right]_P^I = 0. \quad (35)$$

As noted at the beginning of this section, photons do not affect the momentum and energy of ions and atoms but can only change their number densities, we obtain from (34) and (35) the collision terms for ions and atoms due to photon-initiated ionization, as

$$\left[\frac{\delta (n_i f_i^i)}{\delta t} \right]_P^I = \frac{(3 k T_e + 2 \varepsilon_I) R_a \Phi_0 f_{0i}^i}{4 (k T_e)^2 C_v^I} \int K_\nu \left(I_\nu - I_{\nu 0} + \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega, \quad (36)$$

and

$$\left[\frac{\delta (n_a f_a^a)}{\delta t} \right]_P^I = \frac{(3 k T_e + 2 \varepsilon_I) R_a \Phi_0 f_{0a}^a}{4 (k T_e)^2 C_v^I} \int K_\nu \left(I_{\nu 0} - I_\nu - \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega. \quad (37)$$

Finally, substituting (34) into (33) gives the collision model for the electron gas related to radiative ionization, as

$$\begin{aligned} \left[\frac{\delta (n_e f_e^e)}{\delta t} \right]_P^I &= \frac{\mathbf{U}_e f_{0e}^e}{c k T_e} \cdot \int K_\nu \mathbf{l} (I_\nu - I_{\nu 0}) d\nu d\Omega \\ &+ \frac{(3 k T_e - 2 \varepsilon_I - 2 m_e \mathbf{U}_e^2) R_a \Phi_0 f_{0e}^e}{4 (k T_e)^2 C_v^I} \int K_\nu \left(I_{\nu 0} - I_\nu + \frac{1}{c} \bar{\mathbf{V}} \cdot \mathbf{l} I_\nu \right) d\nu d\Omega. \end{aligned} \quad (38)$$

Hence, Eqs. (36) – (38) complete the modelling of the radiation-initiated ionization term in model Eq. (10), and the model can be shown to obey Boltzmann H-theorem.

¹ K. A. Brueckner and S. Jorna, Rev. Mod. Phys. **46**, 325 [1974].

² H. Hora, in "Laser Interaction and Related Plasma Phenomena", edited by H. Schwarz and H. Hora, Plenum Press, New York **1**, 383 [1970], and **2**, 341 [1972].

³ J. W. Shearer and J. L. Eddleman, Phys. Fluids **16**, 1753 [1973].

⁴ J. A. Stamper and D. A. Tidman, Phys. Fluids **16**, 2024 [1973].

⁵ W. M. Manheimer and E. Ott, Phys. Fluids **17**, 1413 [1974].

⁶ W. F. Phillips and V. S. Arpaci, J. Plasma Phys. **7**, 235 [1972].

⁷ J. R. Saraf, Physica **70**, 155 [1973]; Phys. Rev. A **10**, 214 [1974]. Z. Physik A **278**, [1976], in press.

⁸ J. R. Saraf, J. Chem. Phys. **63**, 2577 [1975].

⁹ J. R. Saraf, Phys. Fluids **19** [1976]; Physica **83 c** [1976], in press.

¹⁰ J. H. Clarke and C. Ferrari, Phys. Fluids **8**, 2121 [1965].

¹¹ D. G. M. Anderson, H. Baum, and M. Krook, in "Rarefied Gas Dynamics", edited by L. Trilling and H. Wachman, Academic Press, New York 1969, **1**, 49.

¹² W. F. Phillips and V. S. Arpaci, J. Plasma Phys. **13**, 523 [1975].

¹³ D. H. Sampson, Radiative Contributions to Energy and Momentum Transport in a Gas, Interscience Publ., New York 1965, p. 32–34.